Students' Errors in Compiling Geometric Proof of Gender: A Gender Comparative Study

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Abstract

Mathematical proofing ability is important to learn because it is very influential on the ability of mathematics. The low ability of mathematical proofing is still a problem that is often encountered. Therefore, the low capability of the evidence needs to be explored to know the root of the more detailed problems to be used as initial data so that the solution becomes more precise and effective. Some researchers claim that gender differences also affect mathematical skills. This study aimed to determine the errors and causes in the proof of geometry regarding gender differences. This research was qualitative descriptive research conducted in the Department of Mathematics Education Faculty of Teacher Training and Education, Syiah Kuala University with subject consisting of male and female students. Techniques of collecting data were done through tests and interviews. Data analysis was done based on Miles and Huberman stages, namely data collection, data reduction, data presentation, and conclusions. The error data compiling the geometry proof was analyzed based on Newmann Error Analysis (NEA). The results showed that: 1) mistakes made by male students tend to Process Skill Error; 2) mistakes made by female students tend to Transformation Error; 3) the cause of errors due to the lack of understanding of the concepts (definition, theorem, illustrations, and usefulness) of the students, the lack of logical knowledge and the structure of the method of proof (the way of conclusion either direct or indirect evidence) limitations of language and notation, and lack of care in the settlement procedure.

Keywords: students' errors, geometric proof, gender differences.

Introduction

Learning about mathematical proof has been taught from secondary education to higher education. Learning about mathematical proofs needs to be explained primarily at the university level, this is because most of the material content is related to proof. Moreover, the evidence is the essence of mathematics and this means that one cannot be said to learn mathematics unless he has studied what and how mathematical proofs, the assumption that evidence isn't so necessary in studying mathematics means indirectly separating mathematics from the evidence that is the essence of mathematics itself (Hanna, 2000). Stefanowich (2014) states that proof is a logical line of statements, where one statement results in another report and explains why a statement is true. In this case means that the students are expected to have an adequate understanding of the concept in the proof because in the evidence there is

inter-linkage between concepts in declaring a statement is valid or not, the concept is either a definition, theorem, or lemma. Geometry is one of the courses given at the level of college, especially mathematics education courses. Much of the material contained in high geometry courses is the evidentiary tasks associated with entries, theorems, and the corollary so that the proof activity is the main thing that needs to be mastered and faced by every student in studying geometry.

Therefore, students are expected to be able to master the ability to prove in learning mathematics. Students still experience some severe difficulties in compiling the proof (Martin & Harel, 1989). The mathematical proof is the difficult mathematical concept for students' both to study and to arrange it (Pfeiffer, 2011). So the students' difficulties in preparing the evidence are not enough to see from the proof produced, but also to understand the process of students in making the proof. Based on the observation results obtained data that generally students still have problems in working on the issue proof of geometry, including that is even less precise in selecting the type of evidence and failed in the settlement. This is indeed a motivation for researchers to explore the problems undertaken by students in the mathematical proof.

Every student' can't avoid the difficulty of learning about mathematical proof. It should is realized that in general, the students experience different levels of difficulty in understanding the evidence. The student's mistake in solving the problem of proof can be one of the clues to know how far the students master the material. Therefore, the existence of such errors should are identified and searched for the cause to know the root of the problem in more detail. Thus, information about errors in resolving the problem of proof can be used to provide input in the delivery of solutions. Detailed error analysis is needed so that errors and causes can be found out further to help solve the problem. The stages of error analysis are performed according to Newmann's stages in White (2010), i.e., reading error, comprehension error, transformation error, process skill error, and encoding error. The types of mistakes have classified into five categories which include reading error that is the wrong student in reading and understanding the command questions, and learners misunderstand the symbols in the matter, the comprehension error that learners do not know what is knowns from the problem and do not know what has asked from problem, transformation error is wrong in determining problem-solving strategy and wrong in using formula, process skill error that is error happened at operational algebra and problem solving process error, and encoding error that learners have been able to determine settlement of problem, but learners yet precisely write down the correct procedures and forms of answers.

In addition to errors in the proof, some researchers also stated that gender factors also affect the way to acquire mathematical knowledge. One according to Keitel (1998) shows that gender is a factor that is quite influential in the process of conceptualization. The influence of gender factors in the conceptualization process shows that gender can influence the use of intuition in understanding mathematical concepts. Several studies have examined how gender differences relate to mathematics, male and female learning compared to using variables including innate abilities, attitudes, motivations, talents, and performance (Goodchild & Granholm, 2007). Geary (2000) says girls, in general, are superior in the field of language and writing, while boys are superior in math because of better spatial abilities. Kartono (1970) showed that men tend to be more rational in dealing with problems than female', the male generally has intellectual, thorough thinking abilities while women tend to think real and practical. Therefore, interesting enough to research to see how the role of gender in mathematical proof. Based on some theories that state that gender is also influential in learning mathematics, the researchers wanted to know whether gender differences also affect the ability of proof.

Based on some of the above explanation of the importance of studying evidence in mathematics, it can have concluded that the ability proof influences mathematical ability. The opinions of the experts above also state that proof is a material that is difficult to understand. This is a problem that must be addressed immediately because it affects the mathematical ability. One solution to overcome the problem is by applying the right method in learning proof. Before giving a solution, we should know the problem that students do in compiling the proof by knowing the types of errors and causes so that the solution is applied more precisely and concerning on a real problem. So the purpose of this study is to determine the types of errors and causes of student error in preparing the evidence concerning gender differences.

Research Method
The type of research used was descriptive research with a qualitative approach. This research was conducted in the even semester of academic years 2017/2018 with the subject of research students' of Mathematics Education Faculty of Teacher Training and Education, Syiah Kuala University. Data were collected through tests and interviews and identified according to the Newmann Error Analysis stage which included reading error, comprehension error, transformation error, process skill error, and encoding error. Data analysis using Miles and Huberman consisted of several stages: data collection, data reduction, data presentation, and conclusion drawing. The procedures undertaken in this study were: 1) the researcher conducted the observation by interviewing one of the lecturers of the geometry lecturer, 2) gave test questions to the students, 3) examined and identified the findings of student faults in proving, 4) conducted interviews, 5) and prepared research reports.

Results and Discussion
Here are some descriptions of the findings of errors and causes of mistakes made by students in solving the test questions provided. Six students of three male and three female were selected to have interviewed for representing the type of wrongdoing.

The following is the result of a females' 1st subject (S1F) answer which was wrong in making an illustration and in doing the problem-solving process.
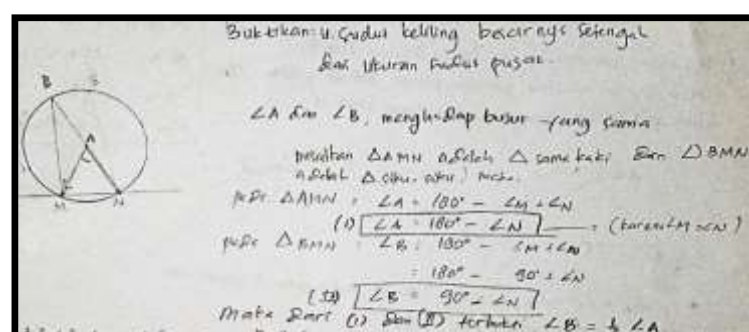

Figure 1. Error Subject 1F

Based on the results of the analysis of test answers and interviews of researchers with S1F students', the information obtained was that the subject 1F made a mistake in the stage of Transformation Error, which was wrong in making illustrations to solve this problem. The students' immediately determined that ΔBMN was right-angled, S1F uses a specific example in the proof. The cause of the errorwasS1F could not make a connection with existing facts, so consequently deliberately set a right triangle to make it more comfortable in the process. Recio and Godino (2001) found that the ability of students' to produce evidence is still insufficient deductively. So it stills proves a statement by using the particular example that doesn't apply in general. Then the 1F subject was also wrong at the process skill error stage, that pointed to

the mistake in the completion operation of $\angle M + \angle N = \angle N$. The correct answer should have been $\angle M + \angle N = \angle 2N$ because $\angle M = \angle N$. The cause of this error was because the subject of 1F did not understand the concept of adding two angles that are congruent and thus wrong in performing its operation. In general, the mistakes made by the 1F subject were concept errors and procedural errors. This finding is supported by the results of Waluyo & Sri (2018) study which stated that a standard error in the problem of proof is rooted in the understanding of the less-than-good concepts of definition, theorem, and lemma.

The following is the result of a males' 1st subject (S1M) error in understanding the relationship between the center angle and the circumferential angle:
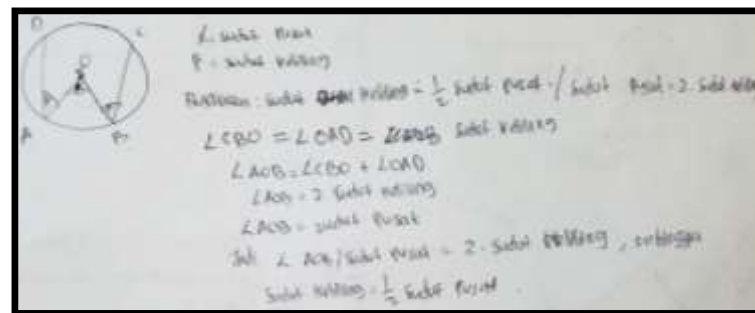

Figure 2. Subject Error 1M

Based on the results of the analysis of test answers and interviews of researchers with S1M students', it was obtained information that S1M mistaken in understanding the purpose of the matter, the subject made two roving angles but not facing the same arc with the central angle. At this stage, it meant that S1M had not been able to identify the premise and its implications and supporting conditions. The error committed S1M was at comprehension error stage. The cause of error due to S1M did not understand the concept of roving angle and center angle. There is in line with the findings made by Harel (1999) the cause of the students' difficulties in proof, namely their lack of understanding of the theorems or concepts and they are wrong in applying them systematically. The next stage was the selection of evidence type. Subject 1M made a mistake on the process skill error stage, which was wrong in performing the problem-solving procedure of $\angle AOB = \angle DAO + \angle CBO$. Based on the interview results, S1M explained that there were two roving angles but not facing the same arc with the center angle. Subject 1M said that $\angle DAO = \angle CBO$ because of the circumference angle, which was done to obtain results $\angle AOB = 2\angle DAO$. Based on the quote of interviews, S1M did not sure with the results of test answers due to incorrect understanding of the meaning of the problem. In general, the mistakes made by subject 1M were the concept errors and procedural errors.

The following is the result of a false females'2 subjects (S2F) answer in linking a theorem to prove:
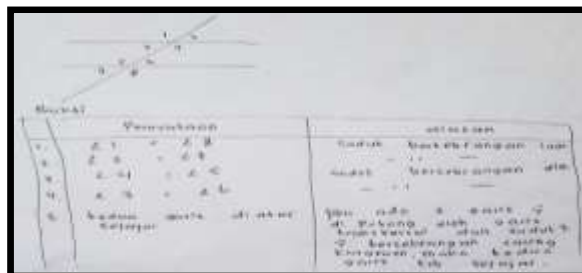

Figure 3. Subject Error 2F

Based on the results of the analysis of the test answers and interviews with the S2F researchers, obtained information that the subject 2F made a mistake in the stage of Transformation Error, namely the selection of evidence that is less precise, students use the type of direct proof with the reason there is a theorem that can assist in the settlement. There means that S2F hasn't understood that the strategy used is correct or not. Due to the improper selection of evidence types, the proofing procedure performed is incorrect. Subject 2F directly uses the opposite angle theorem, and the opposite angle theorem then concludes that the two lines are parallel. There means that the S2F hasn't been able to make a connection between fact and the element of conclusion to be proved, it can see from the student's reply that is still wrong in relating a theorem in conducting the proof. Subject 2F makes a mistake in the process skill error stage, which is wrong in using the related theorems and axioms. Subject 2F doesn't understand the concept of the opposite angle theorem, so the S2F assumes that the given problem is a parallel-line theorem, not an alternate interior angle theorem. In general, the errors made by the S2F are concept errors and procedural errors. Results of research conducted Paduppai and Assagaf (2016) showed the causes of student difficulties in mathematical proof of lack of understanding of mathematical evidence, and lack of understanding of concepts and principles of mathematics.

The following is the result of the male' 2 subjects (S2M) which is wrong in writing the assumption of the conclusion negation.
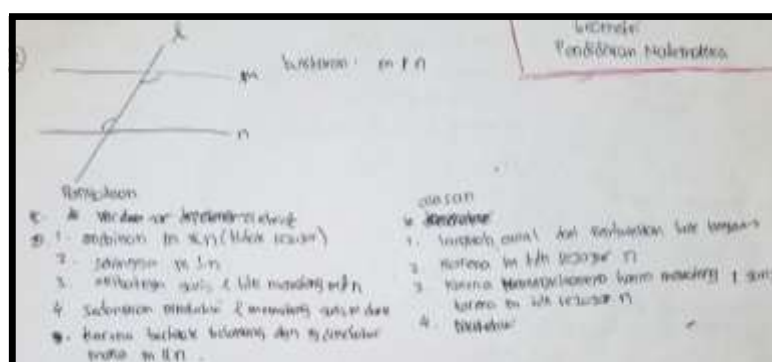

Figure 4. Subject Error 2M

Based on the results of the analysis of the test answers and interview the researchers with S1L students, obtained information that the S2L made a mistake in the Process Skill Error, which is wrong in writing the assumption of the negation of a conclusion. Subject 2L assumes that if $m \nparallel n$ than $m \perp n$ (this is an incorrect assumption), and resulted in that line l not intersect m and n whereas it is known that l intersect m and n into contradictions of the known. The subject S2M can't make a connection between fact and the element of conclusion to be proved, this is seen at the end of the answer which concludes that it is proved if the two lines are parallel. There is a false conclusion, S2M concludes but doesn't comply with the procedure of proof. The cause of the error due to S2M doesn't understand the concept of a parallel line of line and intersect the line. To result in the wrong in writing the assumption of the given loop on the problem. This is also in line with the findings of Stavrou (2014) which sums up the error in the verification of an assumption error; the assumption referred to here relating to the proof of the statement with the structure of the sentence implication. In general, the mistake made by the subject 2M is a conceptual error.

The following is the result of a female' 3 subjects (S3F) who is wrong in understanding the problem and wrong in choosing the type of proof.
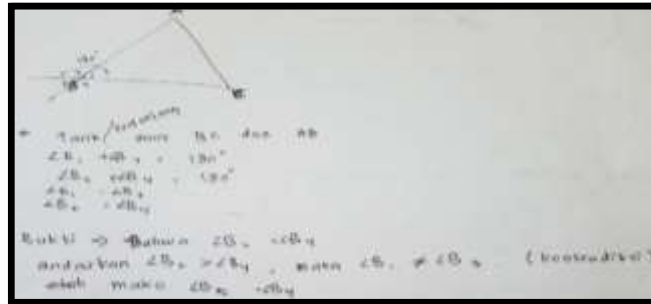
Figure 5. Subject Error 3F

Based on the results of the analysis of the test answers and interviews of researchers with S3F students', it was obtained information that S3F made a mistake in the comprehension error stage.S3F did not understand the meaning of the matter, yet already know what was meant by the outside angle of the triangle but did not understand what should have been proven. It could be seen from the answer where the student proved the vertical angle. The next stage, students experienced an error at the stage of Transformation Error, which was wrong in the selection of types of evidence. Subject 3F had not been able to make a connection between facts with elements of the conclusion to be proved and could have seen in the process done by students' in the proof. The students had not understood the procedure for indirect proof, and the S3F said that for indirect proof by assuming that the statement is false so that a contradiction has been found. Based on the interview also shows that the students' don't understand what the hypothesis and conclusions of a statement are. Based on the excerpt from interviews 3F subjects still encountered an error in the encoding error stage isn't right in writing the procedure of proof and the correct answer. The cause of the error because the 3F subject doesn't understand the concept of direct proof type problem, consequently wrong in understanding the problem and wrong in choosing the type of proof as well as a lack of understanding of mathematical logic so that it still encounters errors in conclusion to the direct or indirect proof. Paduppai and Assagaf (2016) point out the causes of students' difficulties in mathematical proofs: lack of understanding of mathematical proof, and lack of knowledge of mathematical concepts and principles.

The following is the result of the three males' subject (S3L) which is wrong in writing the interior symbol angle that is not adjacent to the exterior angle of the triangle:
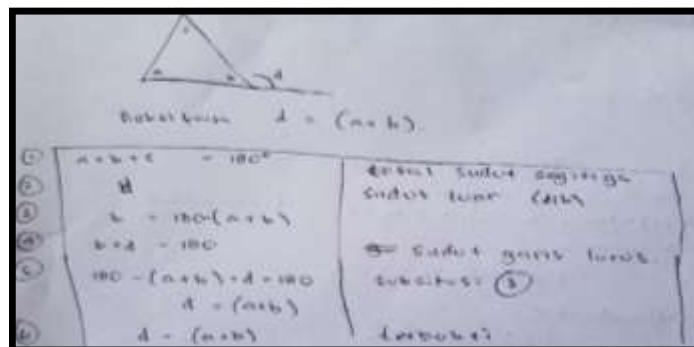

Figure 6. Subject Error 3M

Based on the results of the analysis of test answers and interviews of researchers with S3M students', obtained information that the subject 3M already understand the purpose of the matter, students' already know what is known and what will be proven. At the interview students' can also explain the answer correctly and correctly. Subject 3M made a mistake at the reading error stage is wrong in writing the symbols that

ordered on the matter. Although this procedure is correct because students' are wrong in determining what to prove then the result is also wrong. Subject 3M made a mistake in the encoding error stage, which hasn't been able to write the final answer correctly. Subject 3M has been able to make connections between facts with elements of the conclusion that would be proven although there is still a little mistake. Males' are superior in reasoning, but lack superiority in precision, precision, precision, and thoughtfulness (Krutetskii, 1976).

Errors in compiling geometric proofs by male and female subjects are misunderstanding, error in denying conclusions, failure in the selection of evidence type, error in making an illustration, mistake using theorem in proving, and errors in performing the settlement operation. Errors made tend to comprehension error, transformation error, and process skill error. Female subjects tend to make more mistakes on transformation error while the male subject tends to process skill error. Females' subject to many mistakes when determining a strategy in working on the problem, this is due to lack of understanding of the concept of definition, theorem, or entry. While male' make more mistakes in the process of completion, this is due to the lack of conceptual understanding and lack of meticulous in the proof procedure. Female have only real observation skills, simple analysis, and simple patterns, and are reluctant to try complex calculations, unlike male students having concrete and abstract observational abilities, analysis, syntheses, complicated patterns, generalized conjectures, and test it on the desired answer (Sri, 2013). Male do have a better ability in abstract, but less careful in doing the job. For example, in doing a problem with the exterior triangle angle theorem shows that the male subjects experience an error on the reading error, which is less thorough in writing the symbol in question. While on the subject of female not found an error in the phase of reading error. Here it means that the subject of a female is more accurate in reading the matter. This finding is consistent with Krutetskii (1976) which explains the difference between male and female in learning mathematics, i.e., male superior in reasoning, while female' is superior in accuracy, precision, precision, and thoughtfulness. In general, the mistakes made by students are misconceptions and settlement procedures.

The cause of the student made a mistake in proving not only because of the lack of knowledge of the material content. Sometimes students' know the definition and can explain it informally but cannot use the definition to write evidence. Recio and Godino (2001) found that the ability of students' to produce evidence is still insufficient deductively. So it can't determine valid and legitimate evidence. The overall cause of the error is the lack of understanding of concepts in the form of definitions, illustrations, and usability, the lack of the knowledge of mathematical logic and the structure of the method of proof, as well as the limitations of language and notation. Waluyo and Sri (2018) found a common error in the problem of proof rooted in logical reasoning or logic from students and added to the lack of understanding of the concept of definition, theorem, and lemma. So the students' more memorize the evidence because they don't understand what proof is and how to write it. The lack of knowledge about the theorems and definitions that students' have is also the cause of errors. Results of research conducted Paduppai and Assagaf (2016) showed the causes of students' difficulties in mathematical proof of lack of understanding of mathematical proof, and lack of knowledge of concepts and principles of mathematics.

Conclusions
Based on the results of the error analysis conducted in this study, it can be concluded that some mistakes made by female students' included: comprehension error is a misunderstanding of the problems, and misunderstanding the hypothesis. Transformation error is error determining the type of proof; error makes an illustration, mistake write down the assumption of the negation of conclusion, an error in denying the conclusion. Process skill error is an error using and associating an

existing theorem and an error in showing a contradiction. Encoding error is an error in writing the final answer.

Some mistakes made by male students' include: Reading Error is an error in writing symbols. Comprehension Error is wrong in making illustrations of the center angle and the circumference. Transformation Error is an error in creating illustrations, and error determines the type of proof. Process Skill Error is an error in denying a conclusion, error in writing the assumption of negation conclusion, error in performing the settlement operation. Encoding Error is an error in writing the final answer.

The causes of mistakes made by the subject of female that isn't able to make connections with the facts that exist, don't understand the concept of the sum of two angles are congruent, don't understand the problem and less precise selection of proof type, don't understand the strategy used is correct or not, lack of understanding of mathematical proofs, the lack understanding of concepts and principles of mathematics, don't understanding the concepts for direct proof, and the lack of understanding of mathematical logic so that there is still a mistake in drawing conclusions for both direct and indirect proof.

The cause of mistakes made by male subjects does not understand the concept of the circumference and the center angle, their lack of understanding of the theorems or concepts, incorrectly applying the concepts or theorems systematically, unable to make connections between facts with elements of the conclusion to be correctly proved, not understanding the concepts of parallel lines and intersecting lines, erroneous or inaccurate in writing the angle in the not side by side.

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